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Possible Antihydrogen Trapping Field and Nonneutral Plasma Density Limits

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Abstract. Axisymmetric magnetic field configurations are being considered for trapping antihydrogen atoms that are recombined in Penning traps. As part of an effort to understand the nonneutral plasma confinement properties of axisymmetric configurations, which necessarily have radial field components, plasma confinement within a magnetic field that is purely radial is studied. Various density limits of nonneutral plasmas confined in radial magnetic fields are evaluated and compared to those of plasmas confined in axial magnetic fields.

ANTIHYDROGEN TRAPPING FIELD

A uniform magnetic field can be combined with the field of one or more axisymmetric current loops in order to produce axisymmetric magnetic well configurations. A combination of this type is being considered [1] for trapping antihydrogen atoms recombined in Penning traps such as those designed for obtaining overlap of positron and antiproton plasmas. Overlap of positron and antiproton plasmas may be achieved using either nested Penning traps [2] or by trapping one species in a virtual anode or cathode produced by the other in a magnetic well (this was hypothesized in Ref. [3]). An example configuration calculation for the field of a single current loop combined with a uniform field is shown in Fig. 1(a). The current loop consists of a current of -230 kA (negative according to the right hand rule) at a radius of 10 cm. The uniform field has a magnitude of 2 T and is directed parallel to the z axis. The field configuration illustrated in Fig. 1(a) produces a toroidal magnetic well having a well depth of about 0.5 T. In addition, the outer contour, which is nearly spherical, indicates the presence of an additional non-toroidal 1 T well depth. The electrode configuration in Fig. 1(b) would be for trapping a positron plasma that would form a virtual anode for trapping antiprotons. A disadvantage with the magnetic field configuration is that the field strength vanishes at the center of the toroidal magnetic well. Positron spin flips are possible in regions where the magnetic field strength vanishes [4]. Such Majorana transitions can result in loss of trapped antihydrogen atoms over a time scale determined by the collision rate. Too small a time scale would make it necessary for the magnetic well to have a non-zero minimum. It is possible to eliminate the zero field

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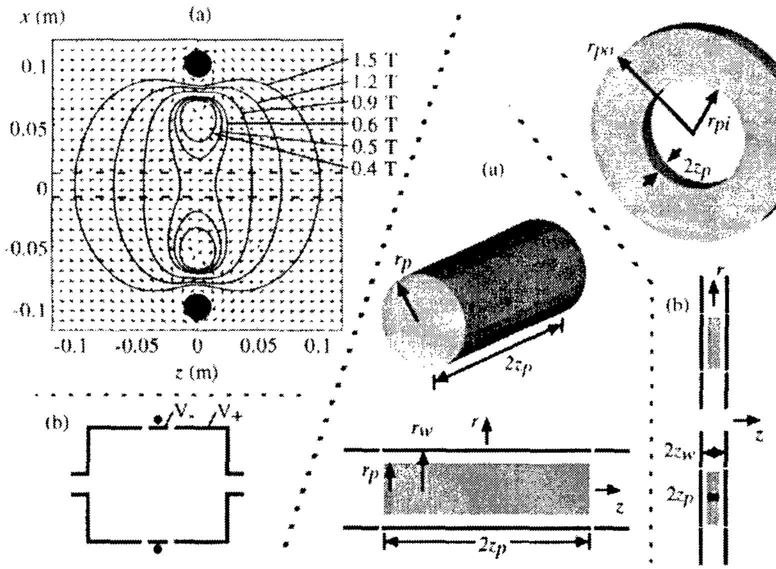

FIGURE 1. (Left) Axisymmetric magnetic field (a) and electrode (b) configurations. The magnetic field configuration is illustrated with a vector plot of the field superimposed on a contour plot of the field strength. The contours are for field strength values of 0.4, 0.5, 0.6, 0.9, 1.2, and 1.5 T. **FIGURE 2.** (Right) Illustration of two axisymmetric nonneutral plasma and electrode configurations. (a) A cylindrical plasma in a uniform axial magnetic field. (b) A washer-shaped annular plasma in radial magnetic field.

location by incorporating an axial current that produces a small azimuthal field outside of it. The axial current could be carried by a wire along the z axis or by a wire mesh, the location of which is illustrated by the dashed lines in Fig. 1(a). The mesh would have the shape of an axisymmetric tube that follows the local magnetic field lines and would have holes through which antihydrogen atoms could pass. To consider the possible use of a current carrying mesh, suppose overlapping positron and antiproton plasmas are confined radially inside the mesh by the axial magnetic field. If the overlap of positron and antiproton plasmas inside the mesh results in recombination, recombined antihydrogen atoms that pass through the mesh holes must lose energy to become trapped within the toroidal magnetic well. This may be possible if the antihydrogen atoms interact with antiprotons or positrons trapped in the magnetic field outside the mesh. Altogether, various possibilities exist for using axisymmetric configurations that produce a magnetic well. Such configurations necessarily include a magnetic field having a radial component.

NONNEUTRAL PLASMA DENSITY LIMITS

Various mechanisms that limit the density of nonneutral plasmas confined in radial and axial magnetic fields (see Fig. 2) are considered using the guiding center approximation in this section (see also Ref. [5]). It is shown with example calculations that the density limits for a nonneutral plasma trapped in a radial magnetic field can be larger than the Brillouin limit. The Brillouin density limit is readily derived by considering a cylindrical single-species plasma centered along the z axis of a cylindrical coordinate system and aligned with a magnetic field. Such a configuration is illustrated in Fig. 2(a). Azimuthal symmetry is assumed, and only radial and axial coordinates (r, z) are referred to. Both the plasma density n and the magnetic field strength B are taken to be static and uniform. The plasma has a radius r_p, is confined within electrodes having an inner wall radius r_w, and is comprised of identical particles, each of charge state Z. The length $2z_p$ of the plasma is assumed to be much larger than the wall radius. Gauss' law provides a suitable approximation for the radial electric field strength at any axial position z that is far from the axial edges of the plasma, $z_p - |z| \gg r_w$. Hereafter, only axial positions z that are far from the axial edges of the cylindrical plasma are considered. The magnitude of the radial component of the electric field within the plasma is $E_r = Zenr/(2\epsilon_0)$, where e is the unit charge and ϵ_0 is the permittivity of free space. The forces on a particle in the plasma consist of a radially outward electric force $F_E = Z^2e^2nr/(2\epsilon_0)$, a radially outward centrifugal force $F_C = mv_\theta^2/r$, and a radially inward magnetic force $F_B = Zev_\theta B$. Here, m and v_θ are the particle mass and azimuthal drift speed. For radial confinement to occur, force balance requires $F_E = F_B - F_C$ or $n = \epsilon_0 B^2(2\chi - \chi^2)/(2m)$ where $\chi = 2mv_\theta/(ZerB)$. The density is maximized when $\chi = 1$, which provides the Brillouin limit,

$$n_B = \frac{\epsilon_0 B^2}{2m}. \tag{1}$$

The Brillouin limit is illustrated by considering a xenon ion plasma in a 0.2 T magnetic field. The Brillouin limit for such a plasma is 8.1×10^{11} m^{-3}.

Confinement in a radial field region is considered, as illustrated in Fig. 2(b). The configuration consists of a static, uniform-density, washer-shaped, single-species nonneutral plasma that is trapped in a radial magnetic field. Azimuthal symmetry is assumed, and the magnetic field in the region of trapped plasma is considered to have only a radial component. The axial plasma width $2z_p$ and the axial separation between electrodes $2z_w$ are both assumed to be much smaller than the radial plasma width $r_{po} - r_{pi}$, where r_{po} and r_{pi} are the outer plasma radius and inner plasma radius, respectively. Gauss' law in planar geometry provides a suitable approximation for the axial electric field strength at any radial position r that is far from the radial edges of the plasma, $r_{po} - r \gg z_w$ and $r - r_{pi} \gg z_w$. Hereafter, only radial positions r that are far from the radial edges of the washer-shaped plasma are considered, and the axial component of the electric field within the plasma is $E_z = Zenz/\epsilon_0$. The associated axially outward electric force on a particle must be balanced by an axially inward magnetic force, $F_B = Zev_\theta B$. Requiring $F_E = F_B$, the particle's azimuthal drift speed must be a linear function of z, $v_\theta = Zenz/(\epsilon_0 B)$. Hence, sheared rotation is a necessary characteristic of the plasma, and a technique (possibly a rotating field technique [6] or a magnetic field that increases with time) may need to be used to control axial plasma expansion.

The Brillouin density limit, Eq. (1), is not applicable to a plasma trapped in a radial magnetic field because a centrifugal force is absent from the axial force balance requirement. Instead, the applied radial electric field must provide a force that balances the centrifugal and magnetic gradient forces on plasma particles as well as the force resulting from the self-consistently produced self-electric field of the plasma. The centrifugal and magnetic gradient forces are first considered by evaluating the radial electric field strengths required to balance each. The radial electric field strength required to balance the centrifugal force is $E_C = F_C/(Ze) = mv_\theta^2/(Zer)$. The largest azimuthal drift speed $v_{\theta,max}$ occurs at z_p, and axial force balance at z_p requires $v_{\theta,max} = Zenz_p/(\epsilon_0 B)$. The radial electric field strength at z_p required to balance the centrifugal force is

$$E_C = \frac{Zemn^2 z_p^2}{\epsilon_0^2 r B^2}. \tag{2}$$

For example, for singly charged xenon ions confined at a density of 1.5×10^{13} m^{-3} in a 0.2 T magnetic field with $r = 25$ cm and $z_p = 0.5$ cm, the electric field strength at z_p required for balancing the centrifugal force is calculated to be 250 V/m.

The radial electric field strength required to balance the magnetic gradient force is $E_M = F_M/(Ze)$. The magnetic gradient force on a plasma particle is $\boldsymbol{F}_M = -\mu \boldsymbol{\nabla} B$ where μ is the magnetic moment associated with a cyclotron orbit [7]. For a magnetic field that has only a radial component in a cylindrical geometry, the condition $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ requires $B = C/r$ where C is a constant. Hence, with $\boldsymbol{\nabla} \rightarrow \hat{\boldsymbol{r}}\partial/\partial r$, and approximating the magnetic moment as $\mu = T_\perp/B$ where $T_\perp$ is the temperature (in energy units) associated with particle motion perpendicular to the magnetic field, $F_M = CT_\perp/(r^2 B)$. With $C = rB$,

$$E_M = \frac{T_\perp}{Zer}. \tag{3}$$

For example, for a plasma of singly charged particles having a temperature of 1 eV and $r = 25$ cm, the electric field strength required for balancing the magnetic gradient force is calculated to be 4 V/m.

The self-electric field of the plasma is now considered, and the centrifugal and magnetic moment forces are ignored. An electric potential well along the magnetic field must be produced such that the plasma is confined radially. Integrating the axial electric field obtained using Gauss' law for a uniform density plasma, the difference in electric potential between the plasma midplane ($z = 0$) and the electrode wall surface is $V = Zenz_p(2z_w - z_p)/(2\epsilon_0)$. The electric potential difference V is approximately that produced by the space charge of the plasma, which tends to shield out the externally applied potential well. Hence, V can be considered an approximate minimum requirement for the magnitude of the externally applied radial well depth for confining the plasma in the limit of zero plasma temperature. [For a finite temperature plasma, V must be increased by an amount much larger than $T_\parallel/(Ze)$, where $T_\parallel$ is the temperature associated with particle motion parallel to the magnetic field.] An associated "confinement voltage" density limit in a radial magnetic field is

$$n_{V,r} = \frac{2\epsilon_0 V}{Zez_p(2z_w - z_p)}. \tag{4}$$

For example, the density limit for a 1 cm wide cold nonneutral plasma of singly charged particles that extends axially to the electrodes, which are separated axially by 1 cm and produce a confinement voltage of 1000 V, is calculated to be 4.4×10^{15} m^{-3}.

The approximate minimum confinement voltage needed for achieving confinement of a uniform cylindrical nonneutral plasma (in the zero temperature limit) along an axial magnetic field is similarly derived. Integrating the radial electric field obtained using Gauss' law, the difference in electric potential between the center of the plasma and the electrode wall inner surface is $V = Zenr_p^2\left[1 + 2\ln\left(r_w/r_p\right)\right]/(4\epsilon_0)$. The electric potential difference V is approximately that produced by the space charge of the plasma, and V can again be considered an approximate minimum requirement for the magnitude of the externally applied well depth. For a given value of V, the confinement voltage density limit for a cold nonneutral plasma in an axial magnetic field is

$$n_{V,a} = \frac{4\epsilon_0 V}{Zer_p^2\left[1 + 2\ln\left(r_w/r_p\right)\right]}. \tag{5}$$

For example, the density limit for a 1.41 cm diameter cold nonneutral plasma of singly charged particles that extends to the inner surface of the electrodes, which produce a confinement voltage of 1000 V, is calculated to be 4.4×10^{15}. The reason the same value is obtained as calculated for $n_{V,r}$ is that Eqs. (4) and (5) give the same value when the plasma extends to the wall and its width is increased by a factor of the $\sqrt{2}$ in Eq. (5).

The electric field produced by a nonneutral plasma is also associated with the possibility of electric breakdown at the inner surfaces of the surrounding electrode walls. Denoting the electric field strength at which breakdown occurs as E_w, Gauss' law can be used to write an associated "wall breakdown" density limit for a washer-shaped plasma in a radial magnetic field:

$$n_{E,r} = \frac{\epsilon_0 E_w}{Zez_p}. \tag{6}$$

According to Ref. [8], field ionization, field desorption, and field evaporation processes at a tungsten anode surface begin to occur at significant rates for electric field strengths roughly between 10 and 50 V/nm. Choosing E_w = 1 V/nm and a plasma width of 1 cm for an example calculation, the resulting density limit is 1.1×10^{19} m^{-3} for $Z = 1$.

For a uniform cylindrical plasma within an axial magnetic field, Gauss' law can be used to write an associated wall breakdown density limit as

$$n_{E,a} = \frac{2\epsilon_0 r_w E_w}{Zer_p^2}. \tag{7}$$

For an example calculation, E_w = 1 V/nm is chosen along with a 2 cm diameter nonneutral plasma that extends radially to electrodes that have a 2 cm inner diameter. For singly charged particles, the associated wall breakdown density limit is the same as calculated for $n_{E,r}$, 1.1×10^{19} m^{-3}, when the plasma extends to the wall and has twice the width.

The Brillouin density limit corresponds to an upper limit for radial force balance to be possible for a uniform cylindrical nonneutral plasma, provided the conditions for reaching

$\chi = 1$ are both possible and desirable. If not, a lower density limit occurs. For a given plasma radius, the density may be limited because the azimuthal drift speed may be limited. The largest drift speed $v_{\theta,max}$ occurs at the radial plasma edge, and the effect results in what may be called the "drift speed" density limit in an axial magnetic field:

$$n_{d,a} = \frac{\epsilon_0 B^2(2\chi_{max} - \chi_{max}^2)}{2m}; \qquad \chi_{max} = \frac{2mv_{\theta,max}}{Zer_pB} < 1. \tag{8}$$

It should be emphasized that Eq. (8) only applies when $\chi_{max} < 1$. Various applications of the drift speed density limit are possible. For example, it is desirable for the speed of newly formed antihydrogen atoms to be sufficiently small for their trapping. If the anti-atoms are formed by antiproton collisions with positronium, the antiproton drift speed (as well as thermal speed) must be sufficiently small. As a second example, it may be desirable for the drift speed to be nonrelativistic for an electron plasma. An approximate density limit is obtained by setting $v_{\theta,max}$ equal to the speed of light, provided $\chi_{max} < 1$.

A drift speed density limit can also be applied to a nonneutral plasma in a radial field. Requiring force balance at z_p, the density limit in a radial magnetic field is

$$n_{d,r} = \frac{\epsilon_0 v_{\theta,max} B}{Zez_p}. \tag{9}$$

For example, consider singly charged xenon ions, and assume that it is desirable for the ion drift speed to be smaller than the thermal speed $\sqrt{T/m}$ (even though larger drift speeds are possible). The associated density limit for $T = 1$ eV ions within an axial width of 1 cm in a 0.2 T radial magnetic field is calculated to be 1.9×10^{12} m^{-3}.

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